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EFFECT OF NORMAL VIBRATIONS OF A FLAT HORIZONTAL HEATER ON THE SECOND BOILING CRISIS

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The effect of normal vibrations of a flat horizontal heater on the second boiling crisis is considered within the framework of the hydrodynamic theory of boiling crises. The critical heat flux is estimated by characteristics of growth of the most dangerous disturbances destroying the liquid–vapor interface. As the vibration intensity increases, the interface can be destroyed either owing to the Rayleigh–Taylor instability or by virtue of parametrically excited disturbances with wavelengths corresponding to resonance zones. In the domain of parameters where the parametric instability in the first resonance zone is the most dangerous factor, it is possible to significantly reduce the critical heat flux, as compared with the value corresponding to the case with no vibrations. With a further increase in vibration intensity, the critical heat flux increases as a whole. The nonmonotonic character of the critical heat flux as a function of vibration intensity allows an effective control of the critical heat flux whose value can be made higher or lower than the value in the case without vibrations.

Key words: *second boiling crisis, vibrations, control of the boiling process.*

Introduction. It is well known that there are two regimes of boiling: bubble boiling and film boiling. Film boiling occurs at high heat fluxes and differs from bubble boiling by the fact that the heater surface is covered by a more or less stable vapor film whose heat-insulating properties significantly reduce heat transfer to the liquid.

The transitions from bubble boiling to film boiling (first boiling crisis) and back (second boiling crisis) occur at certain values of the heat flux, the heat flux corresponding to the first boiling crisis being significantly greater than the heat flux corresponding to the second crisis.

In technological processes, the regime of film boiling is undesirable because of the low heat transfer; therefore, it is important to study various factors that facilitate and prevent its emergence, including the effect of vibrations, which can play an essential role in the behavior of inhomogeneous media [1–5].

The classical hydrodynamic theory of the second boiling crisis, which was developed in [6, 7], is based on the following postulates.

1. The reason for continuous formation and separation of gas bubbles from the liquid–vapor interface in the regime of film boiling is the Rayleigh–Taylor instability.

2. At high heat fluxes, the amount of vapor formed is sufficient to compensate its removal from under the interface with bubbles shed from this surface. The vapor film covering the heater is more or less stable.

3. At moderate heat fluxes, such compensation is impossible, and the development of the Rayleigh–Taylor instability leads to disintegration of the vapor film and to a permanent contact between the liquid and the heater surface.

4. The heat flux corresponding to the second boiling crisis is the least heat flux that ensures the formation of a sufficient amount of vapor per unit time to prevent vapor-film destruction.

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It follows from these postulates that the critical heat flux can be estimated by comparing the vapor-formation rate with a certain typical rate of development of liquid–vapor interface disturbances.

We consider the case where the liquid is heated to the saturation temperature. The phase-change velocity is determined by the heat flux to the liquid–vapor interface, which is assumed to be approximately equal to the heat flux removed from the heater. The fraction of the heat flux caused by the effect of unsteady heat conduction (owing to short-time contacts between the liquid and the heater surface) is not taken into account. For this reason, the actual critical heat flux can be greater than the estimated value. It should be noted, however, that the magnitude of the heat-conducting portion of the heat flux is affected by factors difficult to account, such as the degree of processing of the heater surface, its contamination, etc.; in addition, the heat-conducting portion of the heat flux is normally much smaller than its portion spent on vapor formation.

It was demonstrated by analyzing experimental data in [7] that the liquid–vapor interface is destroyed by the most rapidly growing disturbances; therefore, it was proposed to use their characteristics to calculate the characteristic rate of disturbance development. Following [7], in the case of a flat horizontal heater, we obtain the heat flux corresponding to the second boiling crisis

$$q_{\rm cr2} = (\pi/120) r\rho'' l_d \gamma_d, \tag{1}$$

where r is the specific heat of vapor formation, ρ'' is the mean density of vapor, and l_d and γ_d are the wavelength and growth rate of the most rapidly growing disturbances.

This approach was used to take into account the effect of the heater geometry, pressure, gravitation, etc., on the second boiling crisis (see, e.g., [8]) and yields a fairly accurate estimate for moderate pressures. In the present work, this approach is used to analyze the influence of vibrations. If the field of the gravity force is modulated in time, the Floquet theory predicts the existence of interface disturbances whose growth at the linear stage follows the law $\eta(t) = {\rm e}^{\lambda t} f(t)$, where $f(t)$ is a periodic (with the period of vibrations) function, and the natural analog of the growth rate is the real part of the characteristic measure of disturbances λ.

Equation of Disturbance Evolution. The hydrodynamic theory of the second boiling crisis implies that, because the vapor density is much lower than the liquid density, the governing factor that affects the development of interface instability is the liquid flow rather than the vapor flow. Excess vapor is removed from under the liquid surface by separating bubbles and does not exert any significant effect on the liquid. This is valid to a greater extent for the examined point corresponding to the second boiling crisis, where the rates of vapor formation and vapor removal are commensurable. At the moments when the liquid approaches the heater surface, however, the governing factor is the vapor flow, which moves the liquid backwards, thus, preventing the early onset of the crisis.

By virtue of the assumptions made, the motion of the liquid is described by the Navier–Stokes and continuity equations for a homogeneous incompressible liquid with conventional boundary conditions for the free surface. The equation of evolution of small perturbations Θ of the plane free surface of the liquid in a modulated field of the gravity force for the low-viscosity case was obtained in [1] (see also [2] for the case with no viscosity):

$$\frac{\partial^2\Theta}{\partial T^2} + 4K^2\Delta\,\frac{\partial\Theta}{\partial T} + K(K^2 - 1 + A\cos\Omega T)\Theta = 0. \tag{2}$$

Equation (2) takes into account the contribution of the vapor phase into reduced pressure and the force of inertia, which is usually very small and is taken into account by virtue of tradition only.

Equation (2) is written in dimensionless form: the length and time units are the gravitational-capillary length and time, respectively:

$$d_{\sigma g} = \sqrt{\frac{\sigma}{(\rho' - \rho'')g}}\,, \qquad t_{\sigma g} = \sqrt[4]{\frac{(\rho' + \rho'')^2\sigma}{(\rho' - \rho'')^3 g^3}}\,;$$

the scale of velocity is $v_{\sigma g} = d_{\sigma g}/t_{\sigma g}$. Here σ is the surface tension coefficient, g is the acceleration of gravity, and ρ' is the liquid density.

Equation (2) contains the following dimensionless parameters: dimensionless wavenumber of disturbances K, vibrational overload $A = a\omega^2/g$, dimensionless frequency of vibrations $\Omega = \omega t_{\sigma g}$, dissipative parameter $\Delta = \nu' t_{\sigma g}/d_{\sigma g}^2$ (a is the amplitude of vibrations, ω is the frequency of vibrations, and ν' is the kinematic viscosity of the liquid), and dimensionless time T.

The expression for $Q_{\rm cr2} = q_{\rm cr2}/q_{\rm cr2}^0$, which is the ratio of the critical heat flux to its value in the absence of vibrations, was obtained with allowance for Eq. (1) written in the presence and absence of vibrations:

$$Q_{\rm cr2} = \frac{l_d}{l_d^0}\,\frac{\operatorname{Re}\lambda_d}{\gamma_d^0} = \frac{k_d^0}{k_d}\,\frac{\operatorname{Re}\lambda_d}{\lambda_d^0} = \frac{K_d^0}{K_d}\,\frac{\operatorname{Re}\Lambda_d}{\Lambda_d^0}. \tag{3}$$

(Hereinafter, the superscript 0 refers to the condition without vibrations.) It should be borne in mind that the real part of the characteristic measure of the most rapidly growing disturbances λ_d in the presence of vibrations plays the same role as the growth rate γ_d in the case without vibrations.

In Eq. (3), we performed the transition from the wavelength l_d to the wavenumber $k_d = 2\pi/l_d$ of the most rapidly growing disturbances and used the obvious fact that it is possible to pass from dimensional quantities to their dimensionless analogs in relations of homogeneous quantities. In the absence of vibrations, the dimensionless wavenumber and the growth rate of the most rapidly growing disturbances are $K_d^0 = 1/\sqrt{3}$ and $\Lambda_d^0 = \sqrt{2}/\sqrt[4]{27}$ [which follows from Eq. (2) with no allowance for dissipation]. As a result, we obtain the following expression:

$$Q_{\rm cr2} = \frac{\sqrt[4]{3}}{\sqrt{2}}\,\frac{\operatorname{Re}\Lambda_d}{K_d}. \tag{4}$$

For boiling of water under standard atmospheric pressure in the field of the gravity force with acceleration $g = 9.81$ m/sec^2, the following values of physical parameters are used: $\rho'' = 6 \cdot 10^{-4}$ kg/m^3, $\rho' = 960$ kg/m^3, $\sigma = 0.059$ N/m, and $\nu' = 3 \cdot 10^{-7}$ m^2/sec. In this case, we have $d_{\sigma g} \approx 0.0025$ m, $t_{\sigma g} \approx 0.016$ sec, $v_{\sigma g} \approx 0.16$ m/sec, and $\Delta \approx 7.7 \cdot 10^{-4}$. Thus, the dissipative parameter for water, as well as for many other liquids used in practice, is low.

Equation (2) was used in [3–5] to study the influence of normal vibrations on the Rayleigh–Taylor instability of the free surface of the liquid; the influence of dissipation on parametric instability was also considered there. As far as we are aware, however, the wavelengths and characteristic measures of the most rapidly growing disturbances, which are necessary to estimate the critical heat flux, have not been determined previously.

Because of the low dissipative parameter, the viscous term in Eq. (2) is small for all frequencies of vibrations used in practice and can be omitted (see comments below). In this case, Eq. (2) transforms to the Matthieu equation, which is a particular case of the Hill equation. The Matthieu and Hill equations are standard equations of mathematical physics. The Hill determinant method [9] was used in the present work to determine the characteristic measures of disturbances and their maximum values.

Equation (2) describes two types of instability of the free surface of the liquid: the Rayleigh–Taylor instability and the parametric instability induced by vibrations. Only the Rayleigh–Taylor instability can exist in the absence of vibrations (for $K < 1$), and the parametric instability occurs in the presence of vibrations. The calculations show that the most dangerous factor for low intensity of vibrations is the Rayleigh–Taylor instability; as the intensity of vibrations increases, parametrically excited vibrations with wavelengths corresponding to resonance zones become most dangerous.

Dissipation is known to affect the parametric instability by decreasing (down to zero) the width of the resonance zones (for a fixed amplitude of vibrations) and damping instability in these zones. The effect of this factor, however, depends not only on the magnitude of the dissipative parameter; this effect becomes more pronounced with increasing number of the resonance zone considered and decreases with increasing intensity of vibrations. The calculations show that the growth rate of disturbances in the most dangerous zone depends only weakly on the dissipative parameter (if the latter is sufficiently small, and the frequency of vibrations is not too high), and the number of the zone is uniquely determined by the intensity of vibrations only and increases with increasing the latter.

Formulas of the type (1) for estimating the critical heat flux on the basis of characteristics of growth of the most rapidly growing disturbances were initially proposed for the case without vibrations. In this case, there is a clearly expressed unique maximum of the growth rate (real part of the characteristic measure) of disturbances. In the presence of vibrations, with the Rayleigh–Taylor instability and parametric instability competing in the system, there are several maximums (Fig. 1). The values of the real part of the characteristic measures in certain maximums may be close to each other, whereas the wavelengths can be substantially different. In this case, the liquid–vapor interface is destroyed by disturbances with different wavelengths; hence, Eq. (1) or, more exactly, formula (4) derived from Eq. (1), is inapplicable.

There are some domains of parameters, however, where Eq. (4) is applicable. First, this is the domain of parameters with a clearly expressed main maximum of the growth rate of disturbances, with other maximums being significantly lower (curves 1 in Fig. 1). As a criterion characterizing this case, it is convenient to use the quantity

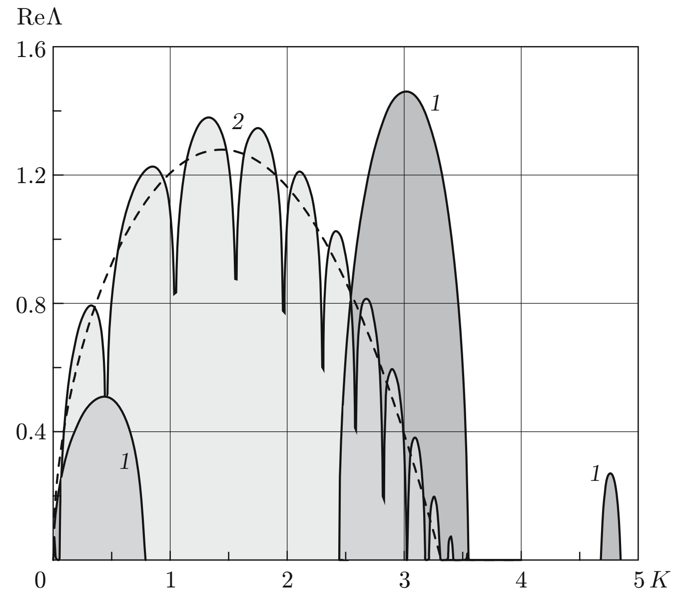

Fig. 1. Real part of the characteristic measure versus the wavenumber of disturbances in the presence of vibrations: $B = 1$ and $\Omega = 10$ (1); $B = 10$ and $\Omega = 10$ (2); the dashed curve shows the WKB approximation for $B = 10$ and $\Omega = 10$.

$\Gamma_\Lambda = \operatorname{Re}\Lambda_{m_2}/\operatorname{Re}\Lambda_{m1}$ inverse to the ratio of the growth rates in the main maximum and in the next maximum in value, which should be much smaller than unity.

Formula (4) is also applicable if the maximums become so condensed that the system does not "feel" the gaps between them. The growth of disturbances occurs in a manner as if there were a single clearly expressed maximum represented by the envelope of the apices of a large number of maximums (curves 2 in Fig. 1). The dashed curve in Fig. 1 shows the data obtained by the Wentzel–Kramers–Brillouin method (WKB approximation). It is convenient to use the relative difference in the wavenumbers in the main and neighboring maximums $\Gamma_K = |K_{m1} - K_{m2}|/K_{m1}$ as a criterion characterizing this case.

Calculation Results. Based on the above-made considerations, the values of the critical heat flux estimated by Eq. (4) are further considered as reliable if $\Gamma_\Lambda \leqslant 0.5$ or $\Gamma_K \leqslant 0.2$. All results in Figs. 2–6 correspond to these values of the criteria Γ_Λ and Γ_K. Thus, the method proposed allows reliable determination of only some segments of the dependence of the critical heat flux on intensity of vibrations, which, however, are sufficient to estimate the character of this dependence as a whole.

The segment $\Gamma_\Lambda \leqslant 0.5$, where the Rayleigh–Taylor instability is the most dangerous factor, corresponds to low intensities of vibrations. The critical heat flux here increases with increasing intensity of vibrations, but its value within the segment considered is only slightly above the corresponding value in the case without vibrations. This is evidenced by the data plotted in Fig. 2 ($B = A/\Omega$ is the dimensionless amplitude of velocity of vibrations).

If the intensity of vibrations is high, the parametric instability becomes the most dangerous factor. In the case of vibrations of a rather high frequency, we can identify the segment $B/\Omega^{1/3} = 0$–2 of the dependence $Q_{\rm cr2}(B/\Omega^{1/3})$ in Fig. 3, where $\Gamma_\Lambda \leqslant 0.5$ and the parametric instability in the first resonance zone is the most dangerous factor. It is seen from Fig. 3 that there are values of parameters for which $Q_{\rm cr2} < 1$, i.e., the critical heat flux can be reduced by using vibrations.

Figure 4 shows the critical heat flux as a function of the frequency of vibrations for a fixed value of the criterion Γ_Λ. In this case, for the prescribed values of Γ_Λ and Ω, there are two values of the critical heat flux (lower and upper branches of the curves in Fig. 4). Different points in the curves correspond to different values of the amplitude of vibrations. This is more clearly seen in Fig. 5, which shows the isolines of Γ_Λ on the plane (Ω, B). The higher the frequency of vibrations, the lower the critical heat flux on the lower branches of curves 1–4; its values can be significantly different from those in the absence of vibrations (the decrease in the critical heat flux is determined by the quantity $1/\Omega^{2/3}$). The critical heat flux at the upper branches of the dependence rapidly increases with

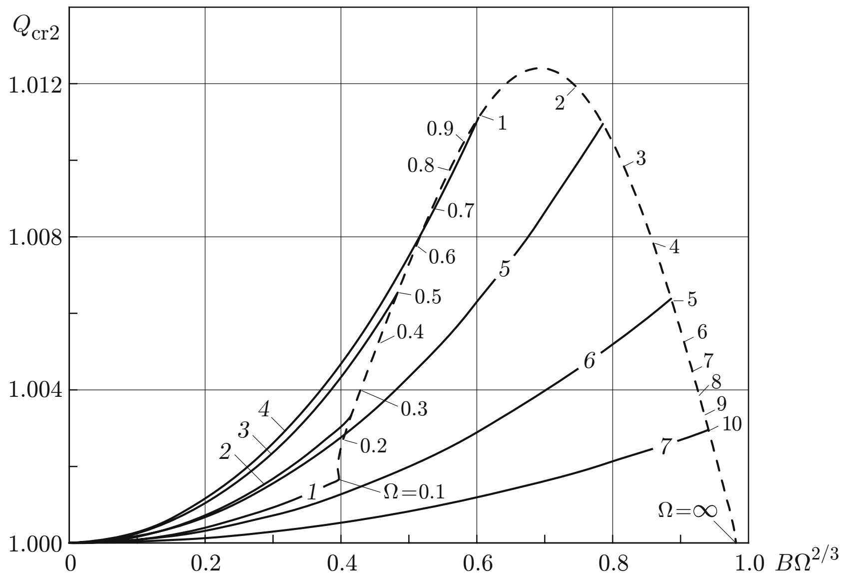

Fig. 2. Critical heat flux versus intensity of vibrations in the region where the Rayleigh–Taylor instability is the most dangerous factor: $\Omega = 0.1$ (1), 0.25 (2), 0.5 (3), 1 (4), 2.5 (5), 5 (6), and 10 (7); the dashed curve shows the dependence $Q_{\rm cr2}(B\Omega^{2/3})$ for $\Gamma_\Lambda = 0.5$.

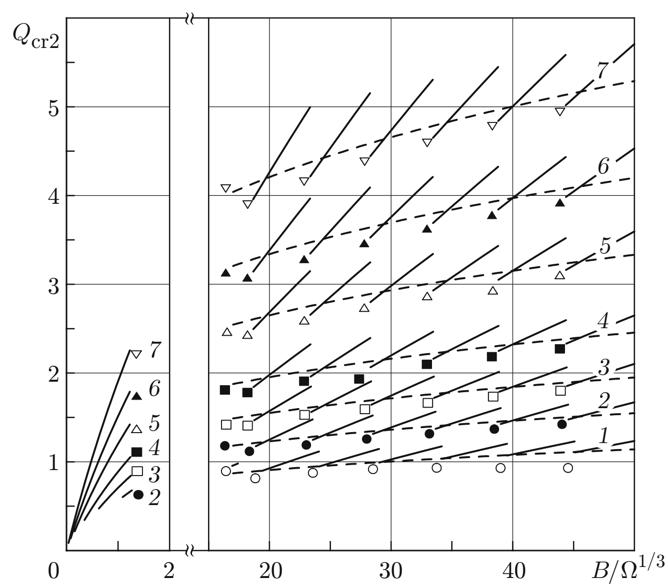

Fig. 3. Critical heat flux versus intensity of vibrations in the region where the parametric instability is the most dangerous factor: $\Omega = 1$ (1), 2.5 (2), 5 (3), 10 (4), 25 (5), 50 (6), and 100 (7); the dashed curves show the data of the WKB approximation.

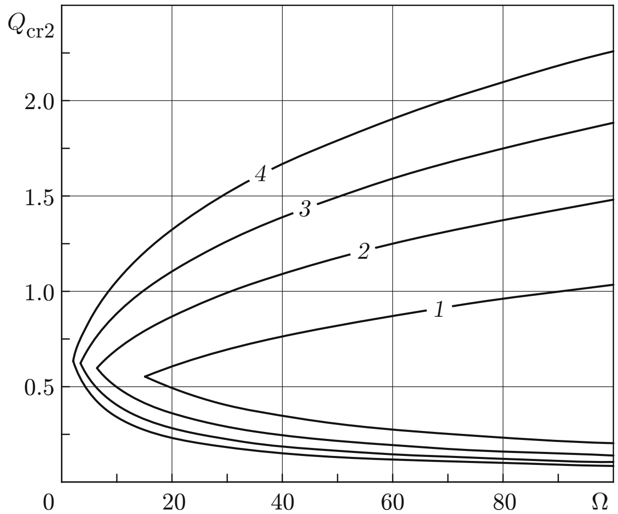

Fig. 4

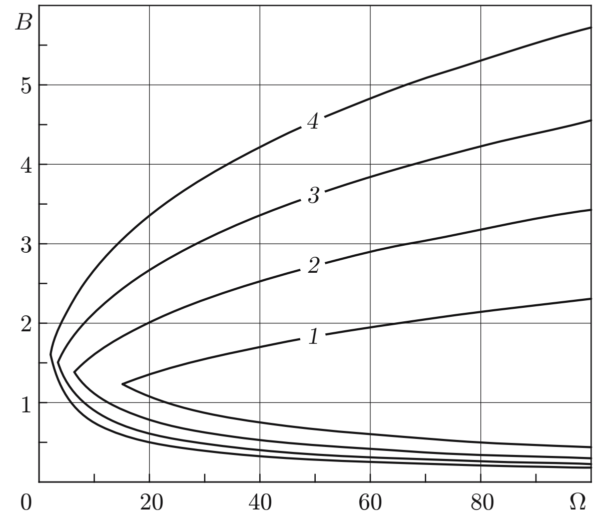

Fig. 5

Fig. 4. Critical heat flux versus frequency of vibrations in the region where the parametric instability in the first resonance zone is the most dangerous factor: $\Gamma_\Lambda = 0.2$ (1) 0.3 (2), 0.4 (3), and 0.5 (4).

Fig. 5. Isolines Γ_Λ on the plane "amplitude–frequency of vibrations" (notation the same as in Fig. 4).

increasing intensity of vibrations. In the case of high-frequency vibrations, the increase in the critical heat flux in this domain of parameters can be significant (upper branches of curves 1–4 in Fig. 4).

Let us consider the results corresponding to the second domain of parameters where Eq. (4) is applicable ($\Gamma_K \leqslant 0.2$), i.e., to the case of condensation of maximums, which occurs for high-intensity vibrations for $B/\Omega^{1/3} \gtrsim 17$ (see Fig. 3). In this case, as the intensity of vibrations increases, the critical heat flux slowly increases in proportion to $\sqrt[4]{A}$ (for $B/\Omega^{1/3} \lesssim 1$, the increase in $Q_{\rm cr2}$ is globally determined by the value of B).

It should be noted that the method used to evaluate the critical heat flux is rather approximate, which is manifested in the "discontinuity" of the curves in Fig. 3. Indeed, the interface is destroyed by a range of disturbances whose characteristics are most close to those of the most rapidly growing disturbances; hence, the real dependence is smoothed. Possibly, the data of the WKB method (see below), which yield a smooth dependence, are more realistic.

Concerning the intermediate domain of parameters where $\Gamma_\Lambda \geqslant 0.5$ and $\Gamma_K \geqslant 0.2$, we can assume that the critical heat flux as a whole also increases with increasing intensity of vibrations.

The critical heat flux is plotted in Fig. 6 as a function of intensity of low-frequency vibrations. In this case, we can again identify a segment of the dependence where the Rayleigh–Taylor instability is the most dangerous factor ($\Gamma_\Lambda \leqslant 0.5$) and also a segment where the parametric instability is the most dangerous factor ($\Gamma_K \leqslant 0.2$). As the frequency of vibrations decreases, the segment of the dependence with $\Gamma_K \leqslant 0.2$ is shifted to the domain of low vibrational overloads and approaches the graph obtained by the WKB approximation.

It follows from Fig. 6 that the critical heat flux can be reduced below its value in the absence of vibrations by using low-frequency vibrations. With a further increase in intensity of vibrations, the critical heat flux slowly increases.

The case with condensed maximums of the characteristic measure of disturbances was considered by the WKB method [10]. The first approximation of the WKB method was used here, which is the method of "frozen" parameters with averaging of the growth rate over the period. The modulated acceleration of the field of the gravity force in Eq. (2), which ignores dissipation, is "frozen." The real part of the characteristic measure is described by the relation

$$\mathrm{Re}\,\Lambda = \pm\frac{1}{\pi}\,\mathrm{Re}\int\limits_0^\pi \sqrt{K(1-K^2)+KA\cos T}\,dT, \qquad (5)$$

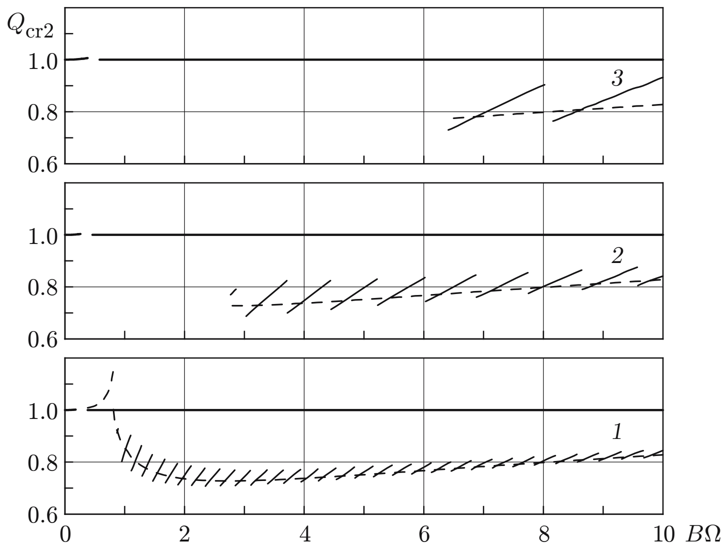

Fig. 6. Critical heat flux versus intensity of vibrations: $\Omega = 0.1$ (1), 0.25 (2), and 0.5 (3); the dashed curves show the data of the WKB approximation.

which can be used to find the wavenumber and the characteristic measure of the most rapidly growing disturbances to finally obtain the critical heat flux.

The dependence $\operatorname{Re}\Lambda(K)$ described by Eq. (5) envelopes the apices of the set of maximums in the graph of the real part of the characteristic measure as a function of the disturbance wavenumber (dashed curve in Fig. 1). Expression (5) offers an adequate description of the actual dependence for either of the situations: $\Omega > 1$ and $B \gg \Omega^{1/3}$, $\Omega < 1$ and $B\Omega \ll 1$, or $\Omega < 1$ and $B \gg 1$.

The critical heat flux as a function of the vibrational overload, which was obtained by the WKB approximation, is plotted by the dashed curve 1 in Fig. 6. The maximum on the dashed curve corresponds to the point with $A \approx 0.804$ and $Q_{cr2} \approx 1.146$, and the minimum occurs at the point with $A \approx 2.767$ and $Q_{cr2} \approx 0.727$.

Two limiting cases (with low and high A, respectively) can be considered within the framework of the WKB approximation:

$$Q_{cr2} = 1 + 3A^2/64, \qquad Q_{cr2} \approx 0.428\sqrt[4]{A}.$$

Conclusions. The study performed allow us to draw the following conclusions. Using vibrations, one can effectively influence the second boiling crisis. As the intensity of vibrations increases, the physical mechanism of destruction of the liquid–vapor interface changes (from the Rayleigh–Taylor instability to the parametric instability), which is responsible for the nonmonotonic character of the critical heat flux as a function of intensity of vibrations. Such a character of this dependence allows one to control the critical heat flux, increasing or decreasing its value as compared to that in the absence of vibrations.

Vibrations of a rather high frequency are preferable to increase or decrease the critical heat flux. Low-frequency vibrations offer only an extremely limited possibility of controlling the critical heat flux; a significant increase is possible only in the case of very high vibrational overloads, which can hardly be achieved in practice.

If it is necessary to retain the film-boiling regime, the critical heat flux should be reduced. This can be done by using the fact that the domain of parameters where the parametric instability on the first resonance zone is the most dangerous factor contains a subdomain where the critical heat flux is lower than that in the absence of vibrations. The decrease in Q_{cr2} is determined by the parameter $1/\Omega^{2/3}$. Hence, a significant decrease in this parameter is only possible in the case of high-frequency vibrations.

If the transition from the film-boiling regime to bubble boiling has to be induced, the critical heat flux should be increased. An increase in the critical heat flux is the basic result of the action of vibrations both in the domain of parameters where the Rayleigh–Taylor instability is the most dangerous factor and in the domain of parameters where the parametric instability is the most dangerous factor. For $B/\Omega^{1/3} \lesssim 1$, the increase in Q_{cr2} is determined by the amplitude B, and the critical heat flux rapidly increases with increasing intensity of vibrations. At the same time, in the case of high-intensity vibrations ($B/\Omega^{1/3} \gg 1$), the increase in Q_{cr2} is determined by the quantity $\sqrt[4]{A}$, and the critical heat flux increases slowly. A particular increase in the critical heat flux being prescribed, it is easier to choose the characteristics of vibrations among high-frequency vibrations with $B \lesssim \Omega^{1/3}$.

Let us consider some examples. In the case of water boiling under standard atmospheric pressure and under standard Earth's gravity, the value $\Omega = 1$ corresponds to the dimensional frequency of vibrations $\omega/(2\pi) \approx 9.95$ Hz, and the value $B = 1$ corresponds to the amplitude of velocity of vibrations $a\omega \approx 0.16$ m/sec. For $\Omega = 100$, i.e., for $\omega/(2\pi) \approx 995$ Hz, a twofold increase in the critical heat flux is reached with $B \approx 4.9$, i.e., with the amplitude of vibrations $a \approx 1.23 \cdot 10^{-4}$ m (segment $B/\Omega^{1/3} = 0$–2 in Fig. 3). For the same frequency of vibrations, a twofold decrease in the critical heat flux is reached with $B \approx 1.08$, i.e., with the amplitude of vibrations $a \approx 2.71 \cdot 10^{-5}$ m.

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